Shutterfly

Shutterfly Photo Story Lesson Plan

Subject: Math
Grade level: 7

Lesson Title: Generate Equivalent Expressions

Common Core/State Curriculum Standards:

- CCSS.Math.Content.7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- CCSS.Math.Content.7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Learning Objectives:
Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Students Learning Targets: *(What will students know and be able to do as a result of this lesson?)*
Students will:

- Generate equivalent expressions using the fact that addition and multiplication can be done in any order (commutative property) and any grouping (associative property).
- Recognize how any order, any grouping can be applied in a subtraction problem by using additive inverse relationships (adding the opposite) to form a sum and likewise with division problems by using the multiplicative inverse relationships (multiplying by the reciprocal) to form a product.
- Recognize that any order does not apply

Instructional Strategies*: (Project-based learning, direct instruction, inquiry-based instruction, cooperative learning, etc.)*
Guided Instruction
Exploration

How Students Will Use Shutterfly Photo Story:
Students will use the Shutterfly Photo App to record findings through the informal exploration. In addition, they will define and illustrate vocabulary words for the lesson. They will also generate and illustrate expressions using the commutative and associative properties. They will include photos and/or drawings of each and utilize the audio record feature to record an explanation.

Required Materials/Lesson Length:
Materials:
Envelopes containing two quadrilaterals and four triangles
iPads with Shutterfly Photo Story app to create their Photo Story

Length:
This lesson will require a full 60-90 minute class period.

Resources: *(Photos, drawings, student created stories; reference books, articles, website URLs, etc. for citation)*
Definitions of the vocabulary words:

- Variable: A variable is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.
- Numerical Expression (in middle school): A numerical expression is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.
    - Statements such as "3+" or "3÷0" are not numerical expressions because neither represents a point on the number line.
- Value of a Numerical Expression: The value of a numerical expression is the number found by evaluating the expression.
    - For example, $13\cdot(2+4)-7$ is a numerical expression, and its value is -5. Note to teachers: Please do not stress words over meaning here; it is okay to use "number computed," "computation," "calculation," etc. to refer to the value
- Expression (in middle school): An expression is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.
- Equivalent Expressions (in middle school): Two expressions are equivalent if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions.
    - This description becomes clearer through lots of examples and linking to the associative, commutative, and distributive properties.
- An Expression in Expanded Form (in middle school): An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in expanded form.
    - A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.
- An Expression in Standard Form (in middle school): An expression that is in expanded form where all like-terms have been collected is said to be in standard form.

Procedures/Activities: *(What will the teacher and students do?) (Prior Knowledge, Opening Activity, Step-by-Step Learning Activities, Closure, Post-Instruction Reflection)*
Exploration Activity:
This exercise requires students to represent unknown quantities with variable symbols and reason mathematically with those symbols to represent another unknown value.

NOTE: Students will record, illustrate, photograph their exploration as they progress through it. Provide each student with an envelope containing two quadrilaterals and four triangles; instruct students not to open their envelopes. Divide students into teams of two to complete A and B below.

- Each envelope contains a number of triangles and a number of quadrilaterals.
- For this exercise, let $\boldsymbol{t}$ represent the number of triangles, and let $\boldsymbol{q}$ represent the number of quadrilaterals.

a. Write an expression, using $\boldsymbol{t}$ and $\boldsymbol{q}$, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

Solution: $\mathbf{3}\boldsymbol{t}+\mathbf{4}\boldsymbol{q}$.

Triangles have **3** *sides, so there will be* **3** *sides for each triangle in the envelope. This is represented by* $\mathbf{3}\boldsymbol{t}$. *Quadrilaterals have* **4** *sides, so there will be* **4** *sides for each quadrilateral in the envelope. This is represented by* $\mathbf{4}\boldsymbol{q}$. *The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.*

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

Solution: $\mathbf{3}\boldsymbol{t}+\mathbf{4}\boldsymbol{q}+\mathbf{3}\boldsymbol{t}+\mathbf{4}\boldsymbol{q}$

$\mathbf{2}(\mathbf{3}\boldsymbol{t}+\mathbf{4}\boldsymbol{q})$

$\mathbf{6}\boldsymbol{t}+\mathbf{8}\boldsymbol{q}$

Discuss the variations of the expression in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

Solution: *Answer depends on the seat size of the classroom. For example, if there are 12 students in the class, the expression would be* $12(3t+4q)$, *or an equivalent expression.*

Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.

Choose one student to open his/her envelope and count the numbers of triangles and quadrilaterals. Record the values of t and q as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

d. Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

$$3t + 4q$$
$$3(4) + 4(2)$$
$$12 + 8$$
$$20$$

There should be 20 *sides contained in my envelope.*

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

Variation #1	*Variation #2*	*Variation #3*
$2(3t+4q)$	$3t+4q+3t+4q$	$6t+8q$
$2(3(4)+4(2))$	$3(4)+4(2)+3(4)+4(2)$	$6(4)+8(2)$
$2(12+8)$	$12+8+12+8$	$24+16$
$2(20)$	$20+12+8$	40
40	40	

My partner and I have a combined total of 40 *sides.*

f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined.

Answer will depend on the seat size of your classroom. Sample responses for a class size of 12*:*

Variation 1	*Variation 2*	*Variation 3*
$12(3t+4q)$	$\overbrace{3t+4q}^{1}+\overbrace{3t+4q}^{2}+\cdots+\overbrace{3t+4q}^{12}$	$36t+48q$
$12(3(4)+4(2))$	$\overbrace{3(4)+4(2)}^{1}+\overbrace{3(4)+4(2)}^{2}+\cdots+\overbrace{3(4)+4(2)}^{12}$	$36(4)+48(2)$
$12(12+8)$	$\overbrace{3(4)+4(2)}^{1}+\overbrace{3(4)+4(2)}^{2}+\cdots+\overbrace{3(4)+4(2)}^{12}$	$144+96$
$12(20)$	$\overbrace{12+8}^{1}+\overbrace{12+8}^{2}+\cdots+\overbrace{12+8}^{12}$	240
240	$\overbrace{20}^{1}+\overbrace{20}^{2}+\cdots+\overbrace{20}^{12}$	240

For a class size of 12 students, there should be 240 sides in all of the envelopes combined

- Have all students open their envelopes and confirm the number of triangles and quadrilaterals match the values of t and q recorded after part (c).
- Have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d).
- Have partners count how many sides they have combined and confirm that number with their answer to part (e).
- Total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

What do you notice about the various expressions in parts (e) and (f)?

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. *The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression* 3t + 4q *in different ways. In part (e),* 3t +3t *is equivalent to* 6t*, and* $4q + 4q$ *is equivalent to 8q.*

There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

Guided Instruction:

Now we will explore how and why we combine numbers and other like terms in expressions. An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include:

324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

Each summand of an expression in expanded form is called a *term*, and the number found by multiplying just the numbers in a term together is called the *coefficient of the term*. Now with the definition of the word *term*, we can explore what it means to "combine like terms" using the distributive property. Remember from the exploration activity, terms sharing exactly the same letter can be combined by adding (or subtracting) the coefficients of the terms:

$$3t + 3t = \overbrace{(3+3)}^{coefficients} \cdot t = 6t, \quad \text{and} \quad 4q + 4q = \overbrace{(4+4)}^{coefficients} \cdot q = 8q.$$

An expression in expanded form with all its like terms collected is said to be in *standard form***.**

Example 1: Any Order, Any Grouping Property with Addition

a. Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

$$5x + 3x = \underbrace{\overbrace{x + x + x + x + x}^{5x} + \overbrace{x + x + x}^{3x}}_{8x} = 8x$$

$$5x - 3x = \overbrace{x + x + \underbrace{x + x + x}_{3x}}^{5x} = 2x$$

Because both terms have the common factor of x, we can use the distributive property to create an equivalent expression.

$$5x + 3x \qquad\qquad 5x - 3x$$
$$(5+3)x = 8x \qquad (5-3)x = 2x$$

Ask students to try to find an example (a value for x) where $5x + 3x \neq 8x$ or where $5x - 3x \neq 2x$. Encourage them to use a variety of positive and negative rational numbers. Their failure to find a counterexample will help students realize what equivalence means. Have students record their discovery in their Photo Story

In Example 1b, students see that the commutative and associative properties of addition are regularly used in consecutive steps to reorder and regroup like terms so that they can be combined. Because the use of these properties does not change the value of an expression or any of the terms within the expression, the commutative and associative properties of addition can be used simultaneously. The simultaneous use of these properties is referred to as the any order, any grouping property.

1. **Find the sum of $2x+1$ and $5x$.**

$(2x+1)+5x$ ***Original expression***

$2x+(1+5x)$ ***Associative property of addition***

$2x+(5x+1)$ ***Commutative property of addition***

$(2x+5x)+1$ ***Associative property of addition***

With a firm understanding of the commutative and associative properties of addition, students further understand that these steps can be combined.

$(2+5)x+1$ ***Combined like terms (the distributive property)***

$7x+1$ ***Equivalent expression to the given problem***

For summary, ask the following questions for reflection:

- Why did we use the associative and commutative properties of addition?
  - *We reordered the terms in the expression to group together like terms so that they could be combined.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same terms as the original expression, just in a different order and grouping.*
- If a sequence of terms is being added, the *any order, any grouping* property allows us to add those terms in any order by grouping them together in any way.
- How can we confirm that the expressions $(2x+1)+5x$ and $7x+1$ are equivalent expressions?
  - *When a number is substituted for the x in both expressions, they both should yield equal results.*

Choose a number, such as 3, to substitute for the value of x and together check to see if both expressions evaluate to the same result. Model this for the students.

Given Expression	***Equivalent Expression?***
$(2x+1)+5x$	$7x+1$
$(2\cdot 3+1)+5\cdot 3$	$7\cdot 3+1$
$(6+1)+15$	$21+1$
$(7)+15$	22
22	

The expressions both evaluate to 22; however, this is only one possible value of x. Challenge students to find a value for x for which the expressions do not yield the same number. Students find that the expressions evaluate to equal results no matter what value is chosen for x.

- What prevents us from using any order, any grouping in part (c) and what can we do about it?

- The second expression, (5a-3), involves subtraction, which is not commutative or associative; however, subtracting a number x can be written as adding the opposite of that number. So, by changing subtraction to addition, we can use any order and any grouping.

a. Find the sum of $-3a+2$ and $5a-3$.

$(-3a+2)+(5a-3)$ ***Original expression***

$-3a+2+5a+(-3)$ ***Add the opposite (additive inverse)***

$-3a+5a+2+(-3)$ ***Any order, any grouping***

$2a+(-1)$ ***Combined like terms (Stress to students that the expression is not yet simplified.)***

$2a-1$ ***Adding the inverse is subtracting***

- What was the only difference between this problem and those involving all addition?
  - *We first had to rewrite subtraction as addition; then, this problem was just like the others.*

Students relate a product to its expanded form and understand that the same result can be obtained using any order, any grouping since multiplication is also associative and commutative.

Example 2: Any Order, Any Grouping with Multiplication
Find the product of 2x and 3.

2x·3=2x+2x+2x=6x

2·(x·3) ***Associative property of multiplication (any grouping)***

2·(3·x) ***Commutative property of multiplication (any order)***

6x ***Multiplication***

Reflection:

- Why did we use the associative and commutative properties of multiplication?
  - *We reordered the factors to group together the numbers so that they could be multiplied.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same factors as the original expression, just in a different order or grouping.*
- If a product of factors is being multiplied, the *any order, any grouping* property allows us to multiply those factors in any order by grouping them together in any way.

Students use any order, any grouping to rewrite products with a single coefficient first as terms only, then as terms within a sum, noticing that any order, any grouping cannot be used to mix multiplication with addition.

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication
Use any order, any grouping to find equivalent expressions.

$$3(2x)$$

$$(3 \cdot 2)x$$

$$6x$$

Ask students to try to find an example (a value for x) where $3(2x) \neq 6x$. Encourage them to use a variety of positive and negative rational numbers because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions. Again, the point is to help students recognize that they cannot find a value—that the two expressions are equivalent. Encourage students to follow the order of operations for the expression $3(2x)$: multiply by 2 first, then by 3.

Closing:

- We found that we can use any order, any grouping of terms in a sum, or of factors in a product. Why?
  - Addition and multiplication are both associative and commutative and these properties only change the order and grouping of terms in a sum or factors in a product without affecting the value of the expression.
- Can we use any order, any grouping when subtracting expressions? Explain.
  - *We can use any order any grouping after rewriting subtraction as the sum of a number and the additive inverse of that number, so that the expression becomes a sum.*
- Why can't we use any order, any grouping in addition and multiplication at the same time?
  - *Multiplication must be completed before addition. If you mix the operations, you change the value of the expression.*

Differentiation: *(Lesson suggestions for enrichment or re-teaching. Scaffolding needed as a result of misunderstandings noted during formative assessment.)*
TBD

Special Education/ESL Accommodations & Modifications:
Allow students to work in small groups or partners
Provide adult support as needed

Extensions: *(Additional activities, follow-up lesson ideas, how the Photo Story book will be shared)*
Have students share their story books in small groups. Encourage discussion and explanations of the examples each student displays in their book.

Assessment: *(How will you determine if students have met the lesson objectives? How will your students know if they have successfully met the lesson objectives? Incorporate formative as well as summative assessments – rubrics, etc.)*
As this is an informal exploration activity, no assessment is required. Teacher will monitor class and group discussions and provided redirection and clarification as needed.